Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	7	6	/	0	1	Signature	

Paper Reference(s)

6676/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Wednesday 3 February 2010 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
Total	

Materials required for examination
Mathematical Formulae (Green)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1.

$$\frac{dy}{dx} = x^2 + 2\sin y$$

It is given that $y = 1$ at $x = 0$.

Use the approximation $\frac{y_1 - y_0}{h} = \left(\frac{dy}{dx}\right)_0$ with a step length of $h = 0.1$ to find estimates of y at $x = 0.1$ and at $x = 0.2$, giving your answers to 4 decimal places.

(5)

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Question 1 continued

(Total 5 marks)

Q1

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2. Given that $y = x^3 \ln x$,

(a) find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$.

(5)

(b) Find the Taylor series expansion of $x^3 \ln x$ in ascending powers of $(x-1)$ up to and including the term in $(x-1)^3$.

(3)

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Question 2 continued

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Question 2 continued

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Question 2 continued

(Total 8 marks)

Q2

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3. (a) Use De Moivre's theorem to show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

(5)

(b) Hence find the two positive solutions of

$$32x^5 - 40x^3 + 10x + 1 = 0,$$

giving your answers to 3 decimal places.

(6)

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Question 3 continued

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Question 3 continued

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Question 3 continued

(Total 11 marks)

Q3

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4. For $n \in \mathbb{Z}^+$, show, using mathematical induction, that

(i) $$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 & 0 \\ n & 1 & 0 \\ n(n+2) & 2n & 1 \end{pmatrix},$$

(5)

(ii) $(2^{3n+1} + 5)$ is divisible by 7.

(5)

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Question 4 continued

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Question 4 continued

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Question 4 continued

(Total 10 marks)

Q4

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5. Given that

$$\mathbf{a} = \mathbf{i} + 7\mathbf{j} + 9\mathbf{k} \quad \text{and} \quad \mathbf{b} = -\mathbf{i} + 3\mathbf{j} + \mathbf{k},$$

(a) show that $\mathbf{a} \times \mathbf{b} = c(2\mathbf{i} + \mathbf{j} - \mathbf{k})$, and state the value of the constant c.

(2)

The plane Π_1 passes through the point (3, 1, 3) and the vector $\mathbf{a} \times \mathbf{b}$ is perpendicular to Π_1.

(b) Find a cartesian equation for the plane Π_1.

(2)

The line l_1 has equation $\mathbf{r} = \mathbf{i} - 2\mathbf{k} + \lambda\mathbf{a}$.

(c) Show that the line l_1 lies in the plane Π_1.

(2)

The line l_2 has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \mu\mathbf{b}$.
The line l_2 lies in a plane Π_2, which is parallel to the plane Π_1.

(d) Find a cartesian equation of the plane Π_2.

(2)

(e) Find the distance between the planes Π_1 and Π_2.

(3)

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Question 5 continued

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Question 5 continued

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Question 5 continued

(Total 11 marks)

Q5

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6.

$$\mathbf{M} = \begin{pmatrix} 11 & -5\sqrt{3} \\ -5\sqrt{3} & 1 \end{pmatrix}$$

Given that λ_1 and λ_2 are the eigenvalues of **M** and $\lambda_1 > \lambda_2$,

(a) show that $\lambda_1 = 16$ and find the value of λ_2.

(4)

(b) Find eigenvectors corresponding to the eigenvalues λ_1 and λ_2.

(4)

Given that there is an orthogonal matrix **P** such that $\mathbf{P}^{-1}\mathbf{MP}$ is the diagonal matrix **D**, where $\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$,

(c) find the matrix **P**,

(2)

(d) verify that $\mathbf{P}^{-1}\mathbf{MP} = \mathbf{D}$.

(4)

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Question 6 continued

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Question 6 continued

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Question 6 continued

(Total 14 marks)

Q6

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7. The point P represents the complex number z in an Argand diagram.
Point P moves on the curve C given by the equation

$$|z-4+4\mathrm{i}| = 2|z-1+\mathrm{i}|$$

(a) Show that C is a circle whose equation may be written $|z| = k$, giving the exact value of k.

(5)

(b) Draw an Argand diagram showing the circle C and the points representing the complex numbers $1 - \mathrm{i}$ and $4 - 4\mathrm{i}$.

(3)

(c) For the points on the circle C, find the maximum and minimum values of $|z-4+4\mathrm{i}|$.

(3)

The transformation T from the z-plane to the w-plane is given by $w = z + \frac{8}{z}$.

(d) Show that T maps the curve C onto a line segment in the w-plane and define this line segment by giving its equation and the coordinates of its end points.

(5)

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Question 7 continued

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Question 7 continued

Q7

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

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